

PAPER

An accuracy evaluation of the sound power measurement by the use of the sound intensity and the sound pressure methods

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Abstract: Because of its plane-wave assumption at measurement points, the pressure method is in essence an approximation in determining the sound power of a source. When using the sound intensity method, the measurement errors are theoretically due only to the finite number and locations of the measurement points. In the present paper, the degrees of measurement errors of the two methods are evaluated using a sound source model that comprises multiple point-like spherical sources placed above an infinitely large rigid floor. Using such a sound source model, it is possible to evaluate the measurement accuracies of the sound pressure and the sound intensity methods because the radiated sound power is exactly calculated for arbitrary distributions of point-like sources. By randomly changing the distribution and phases of the point-like sources, it is possible to evaluate the statistical aspects of errors such as mean, standard deviation, maximum and minimum errors. One application of this source model is a comparison of the sets of measurement points proposed by standards. A new set of measurement points is also proposed.

Keywords: Sound power, Measurement method, Sound intensity, Sound pressure

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1. INTRODUCTION

The methods of sound power measurement are standardized by ISO [1–6]. These methods are based on either the sound pressure measurements (3740 series [1–3]) or on the measurements of sound intensity (9614 series [4–6]). Also, we can classify the sound power measurement methods by the measurement environment: the anechoic (or hemi-anechoic) room and the reverberant room. This paper analyses theoretically the measurement accuracy of sound power measurement methods in a hemi-anechoic room that is used more often than an anechoic room.

When using the sound intensity method, the sound intensities are measured at various discrete points on the measurement hemisphere or other types of surfaces surrounding the sound source. The measurement error decreases with the increasing number of measurement points and in the limit becomes infinitely small provided that the sound intensity measurement itself is error-free. In the pressure method, however, it is assumed that the wave

radiated by the source satisfies the plane wave condition at the measurement point and the sound intensity normal to the measurement surface is calculated from a simple relationship between the sound pressure and sound intensity. These assumptions are only approximate and can cause large errors in intensity determinations. In summary, there are two types of sound power measurement errors: the finite number of the measurement points and the plane wave assumption mentioned above, which is rarely satisfied.

It is almost impossible or at least very difficult to evaluate the accuracy of the sound power measurement methods by the pressure and the sound intensity experimentally because the uncertainties of the actual sound power measurement are too large for the purpose of the evaluation of the method. Therefore, the evaluation of the accuracies of the two methods have to be done theoretically or numerically. A suitable sound source model for this purpose is a box (parallelepiped) consisting of five panels, which is placed on a rigid floor in a hemi-anechoic room

(in some ISO standards, a parallelepiped is implicitly assumed as the shape of the sound source). Calculations of the vibration of the panels due to an assumed excitation force can be made by the FEM (finite element method) and the sound pressures at any points on the hemispherical surface surrounding the source can be obtained using the boundary element method (BEM). However, accuracies of the FEM and BEM employed must be evaluated very carefully to keep them on the order of 0.1 dB, which is not at all an easy task.

To avoid the above-mentioned problems, we propose a different sound source model that consists of multiple point-like sources with random mutual phase relationships. These sources are randomly distributed inside an imaginary parallelepiped placed on a rigid floor. The sound field generated by these sources can be calculated exactly by closed formulas, so that the true radiated sound power and pressures at any points in the space can be determined and used to evaluate the accuracy of the pressure and the intensity methods. Hundreds of trials with random positions and phase relations of the point sources will give the statistical values of the measurement errors such as averaged error and standard deviation of the error.

2. A PROPOSED SOUND SOURCE MODEL

In the case of the intensity method, the measurement accuracy depends on the unevenness of the outward normal intensity on the measurement surface. If a sound source is more directional, an error due to the inadequacy of sampling may become larger. In the case of the pressure method, the reactivity of the sound field is an additional factor to the causes of the measurement error. A sound source that consists of two 180 degree out-of-phase point sources is an extreme case that produces a significantly reactive sound field near the source. One more factor must be taken into account is the direction of the sound source relative to the geometry of the measurement points. In ISO 3745 [2], the sound source is implicitly assumed to have a shape of a rectangular box, and a hemi-spherical measurement surface is recommended as shown in Fig. 1.

This paper, considering the above mentioned factors, proposes a sound source that consists of multiple point-like sources (very small but finite spheres in breathing modes), which are distributed inside an imaginary box. Both the positions inside the imaginary box and the phases of the velocities of point-like sources are randomly set in each trial. The direction of the imaginary box with respect to the x-axis is also randomly set in each trial. In this model, it is assumed that all point-like sources have the identical radii and the velocity amplitudes.

Using this sound source model, the trial is repeated, for example, 1,000 times. Since the true radiated power and sound pressure at any point in the hemisphere are

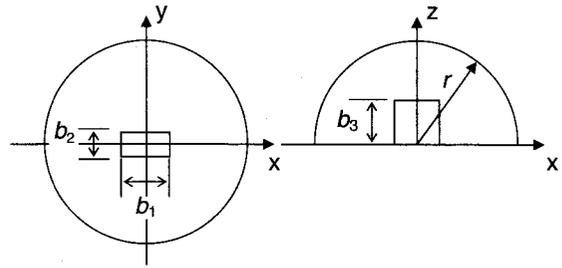


Fig. 1 The imaginary box and the measurement hemisphere.

calculated numerically, the errors of the pressure and the intensity methods can be obtained for each trial. Then, the statistical values of errors such as the mean, standard deviation, maximum, and minimum values can be obtained from these trials. The number of the point-like sources can be varied (from one to any finite number). Also, the number of frequency components, radiated by all point-like sources, in each 1/3 octave band can be varied. In one trial, the same source strength distribution and the same phase relationships among the point-like sources are applied to all frequency components in each 1/3 octave band. This kind of assumption may be applicable to sound sources like loudspeakers with ideally rigid diaphragms and wide reproduction ranges. However, those of actual sound sources (for example, vibrating panels) are not identical but change in some degree in each 1/3 octave band (see, for example, reference [9]).

3. THE CALCULATION OF TRUE AND MEASURED RADIATED POWERS

For two point-like sources with radius a ($ka \ll 1$), the self (Z_{11}, Z_{22}) and mutual (Z_{12}, Z_{21}) radiation impedances (in Ns/m) are given, respectively, by [7,8]:

$$Z_{11} = Z_{22} = 4\pi a^2 \rho c (ka)^2 (1 + j/ka) \quad (1)$$

and

$$Z_{12} = Z_{21} = 4\pi a^2 \rho c (ka)^2 [(\sin kd)/kd + j \cos(kd)/kd] \quad (2)$$

where d is the distance between the point-like sources, ρ is the air density, c is the sound velocity in the air, and k is the wavenumber. Then, the total radiated power is given by:

$$W = R_{11}|v_1|^2/2 + R_{22}|v_2|^2/2 + R_{12}|v_1 v_2| \cos \varphi_{12} \quad (3)$$

where $|v_1|$ and $|v_2|$ are the velocity amplitudes of the 1st and 2nd point-like sources, respectively, R_{11} and R_{22} are the self radiation resistances, R_{12} and R_{21} ($R_{12} = R_{21}$) are the mutual radiation resistances given by the real parts of Eq. (2), and φ_{12} is the phase difference between v_1 and v_2 . Similarly, for $2K$ point-like sources including the mirror images, the total radiated power into the hemi-infinite space is given by:

$$W_T = \left[\sum_{i=1}^{2K} R_{ii} |v_i|^2 / 2 + \sum_{i=1}^{2K-1} \sum_{j=i+1}^{2K} R_{ij} |v_i v_j| \cos \varphi_{ij} \right] / 2 \quad (4)$$

The parameters with subscripts i and j in Eq. (4) are those of the i -th and j -th point-like sources, respectively. If the source vibrates at various frequencies, the total power is given by the summation of those components.

The total sound pressure generated by the $2K$ point-like sources at the m -th measurement point is given by [8]:

$$p_m = \sum_{i=1}^{2K} \rho c v_i \frac{jka}{1 + jka} \frac{ka}{kd_{im}} e^{-jk(d_{im}-a)} \quad (5)$$

where d_{im} is the distance between the i -th point-like source and the m -th measurement point. Then, the estimated sound intensity from the pressure at the measurement point is given by:

$$I_m^P = |p_m|^2 / 2\rho c \quad (6)$$

The total estimated power determined by the pressure method is given by:

$$W_P = \sum_{m=1}^M I_m^P \Delta S_m \quad (7)$$

where M is the number of measurement points and ΔS_m ($= 2\pi R^2 / M$) is the measurement area that each measurement point is supposed to cover.

Since the pressure at arbitrary points are calculated by using Eq. (5), true sound intensities at measurement points, that are normal to the spherical measurement surface, can be also obtained. First, the particle velocity normal to the measurement surface is approximated by:

$$v_{nm} = -\frac{1}{j\omega\rho} \frac{(p_{m1} - p_{m2})}{\Delta R} \quad (8)$$

where p_{m1} and p_{m2} are the pressures at the m -th measurement. They are separated with distance ΔR ($k\Delta R \ll 1$) and must be on a line normal to the measurement surface. Then, the sound intensity is obtained by:

$$I_m^I = \text{Re}\{[(p_{m1} + p_{m2})/2]v_{nm}^*\}/2 \quad (9)$$

and the total radiated power is given by:

$$W_I = \sum_{m=1}^M I_m^I \Delta S_m \quad (10)$$

W_P and W_I are the powers determined by the pressure and sound intensity methods, respectively.

The accuracies of the program to calculate W_T , W_P and W_I can be checked by assuming that the point-like sources are concentrated at the origin. It was confirmed that the errors were on the order of 10^{-4} dB for the case with $a = 0.000001$ m, $R = 1.0$ m, and $\Delta R = 0.000001$ m.

Errors of W_P and W_I relative to the true radiated power W_T will be evaluated in the following sections for various measurement conditions.

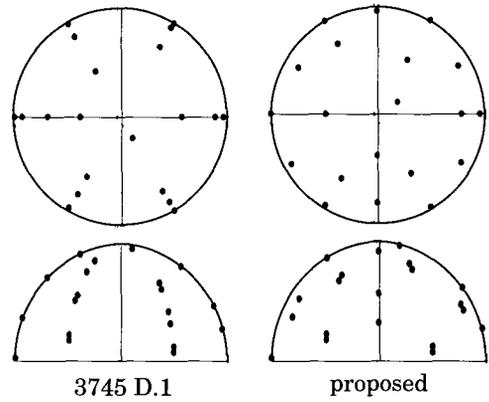


Fig. 2 The horizontal (top) and vertical (bottom) plane distributions of the twenty measurement points defined by Table D.1 in ISO 3745 (left) and its modified version (right).

4. THE EVALUATION OF THE PROPOSED SOURCE MODEL

First, the effects of some parameters of the proposed sound source model on the statistical values, such as the number of the point-like sources, and the number of trials, will be evaluated. For this purpose, the geometry of the twenty measurement points given by Table D.1 in ISO 3745 [2] is used as the standard measurement point localization. The distribution of the measurement points on the horizontal plane is shown by the left-side chart in Fig. 2. One feature of this measurement point geometry is that the heights of measurement points are increasing from $0.025R$ to $0.975R$ with $0.05R$ steps. In addition, the highest measurement point is slightly off the zenith. The right-side chart in Fig. 2 shows a proposed set of measurement points that will be discussed in section 6.

The values of source parameters and the measurement surface used for the evaluation of the proposed sound source model and measurement point locations, if not otherwise specified, are shown in Table 1. The measurement surface with the radius $R = 1$ m is employed following ISO 3745 [2], which requires that R must be greater than or equal to (1) twice the maximum dimension of the box, (2) quarter wavelength of the lowest frequency of interest, and (3) 1 m. It should be noted that the requirement (2) is satisfied from 86 Hz and higher.

The errors in each 1/3 octave band are evaluated in terms of (1) mean error, (2) standard deviation of errors, (3) maximum error, and (4) minimum error. With the number (N) of repetition of calculations and the number of components L in each band, the mean error (ME) and the standard deviation (SD) for each band are given, respectively, by:

Table 1 The values of source parameters and the measurement surface used (if not otherwise specified) for the error evaluation.

Parameters	Values
Number of the point-like sources	$K = 32$
Number of line spectra in each 1/3 octave band	$L = 30$
number of repetition	$N = 1,000$
Dimensions of the imaginary box	$b_1 = 0.3 \text{ m}, b_2 = 0.4 \text{ m}, b_3 = 0.5 \text{ m}$
Radius of the measurement surface	$R = 1 \text{ m}$

$$ME_{I,P} = \frac{1}{N} \sum_{n=1}^N E_{I,P}(n) \quad (11)$$

where

$$E_{I,P}(n) = 10 \log \left[\frac{\sum_{l=1}^L W_{I,P}(l)}{\sum_{l=1}^L W_T(l)} \right] \quad (12)$$

and

$$SD_{I,P} = \sqrt{\frac{1}{N} \sum_{n=1}^N [E_{I,P}(n) - ME_{I,P}]^2} \quad (13)$$

$E_I(n)$ and $E_P(n)$ are errors in dB of the n -th trial obtained by the intensity and the pressure methods, respectively. $W_I(l)$, $W_P(l)$ and $W_T(l)$ are the powers of the l -th components in the band obtained by the intensity method, pressure method, and the theory, respectively. The maximum and minimum values are found from $E_I(n)$ and $E_P(n)$, $n = 1, 2, \dots, N$.

4.1. The Effect of the Number of Point-Like Sources (K)

The results of errors (mean, standard deviation, maximum, and minimum values) are shown in Fig. 3 for the cases with $K = 2, 16, 32$ (it should be reminded that the total number of point-like sources including the mirror image sources is $2K$). The left and right column charts are those of the intensity and pressure methods, respectively. First, we will discuss the results generally. The mean values of the intensity method are between -0.1 dB and 0 dB. On the other hand, those of the pressure method are always positive and stay between 0 dB and 0.2 dB except those below 100 Hz. The standard deviations of the intensity and the pressure methods have similar frequency dependence except for the low frequency range. The maximum standard deviation takes a maximum value of approximately 0.7 dB at 2 kHz. As expected, the pressure method gives significantly larger maximum errors in the low frequency range. Both methods give approximately the

same minimum errors. Both methods show the same behavior of errors above 1 kHz, where the errors are determined mostly due to the spatial sampling inadequacy. In the low frequency range, however, the pressure method produces large errors compared to the intensity method due to the assumption given by Eq. (6).

The effect of the number of point-like sources on the errors is small except for the low frequency range of the pressure method. In the low frequency range, two point-like sources can produce very reactive sound field if they are out-of phase with each other. As the number of the point-like sources increase, chances to produce reactive sound field become lower. As an extreme case, $K = 2$ must be taken into account for the check of the errors in the low frequency range. In the high frequency range, the standard deviation of errors increases with increasing K , possibly because the large number of point-like sources can produce highly directional radiation patterns. However, the standard deviations stay approximately the same even though K is increased from 16 to 32 , indicating that $K = 16$ will be enough for the evaluation of the errors in the high frequency range.

4.2. The Effect of the Number of Line Spectra (L) in Each 1/3 Octave Band

Since the standards mostly require measurements of radiated powers in 1/3 octave bands, the number of spectral components in 1/3 octave bands are varied and its effects are evaluated. Since different frequency components do not interact with each other in the radiated sound power, the radiated power in each 1/3 octave band is simply given by the summation of powers of components in that band.

Effects of the number of line spectra in each octave band on the mean errors and the standard deviations are shown in Fig. 4 for the cases where $L = 1, 2, 10, \text{ and } 30$. The differences among errors are mostly seen in the high frequency bands. Errors are significantly reduced if L increased from 1 to 2 . The differences are negligible between the cases with $L = 10$ and $L = 30$. If there are 3 or 4 evenly spaced components in 1/3 octave bands, the source may be treated as a broadband source in the radiated power measurement. However, we would like to point out that this conclusion is valid only with the assumption that the same source strength distribution and the phase relationships among the point-like sources are applied to all frequency components in each 1/3 octave band in one trial. If some degrees of changes are introduced, this conclusion may change.

4.3. The Effect of the Number of the Calculation Repetition (N)

The effects of the number of repetitions (N) on the

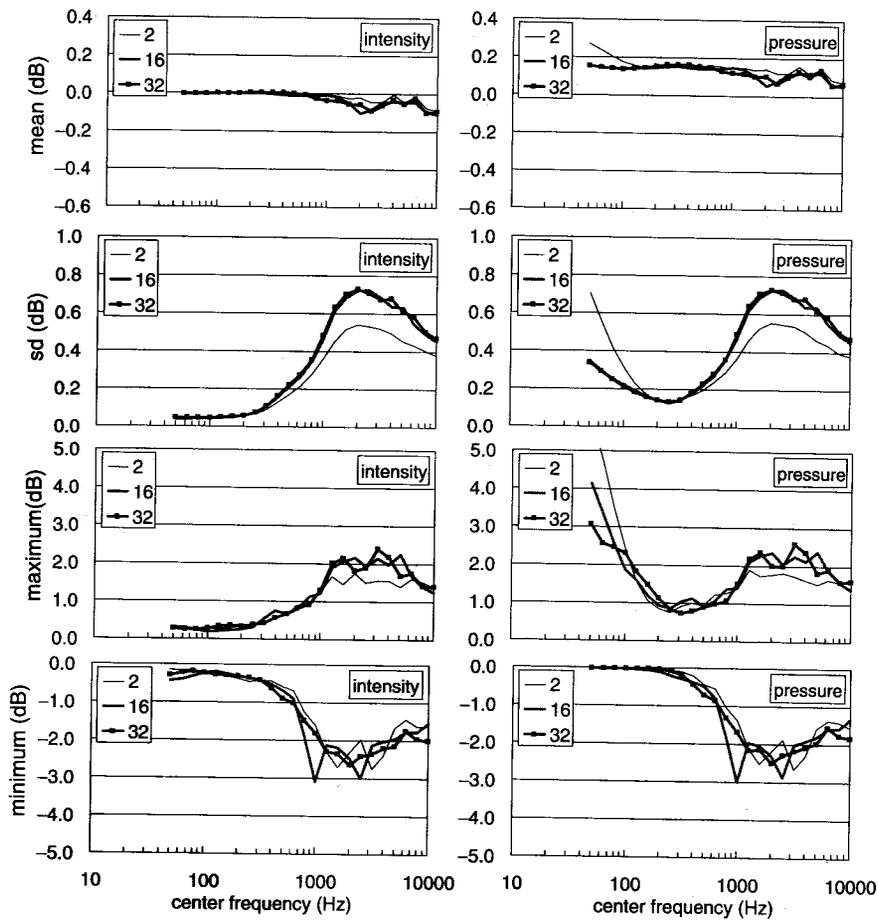


Fig. 3 The dependence of the mean, standard deviation, maximum, and minimum values of errors on the number of point-like sources (K). The left and right columns are for the intensity and the pressure methods, respectively.

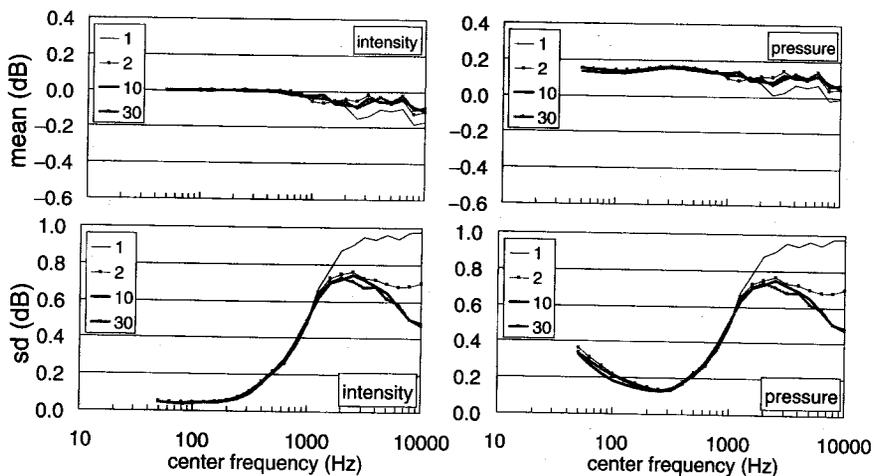


Fig. 4 The dependence of the means and standard deviations of errors on the number of line spectra (L) in each 1/3 octave band.

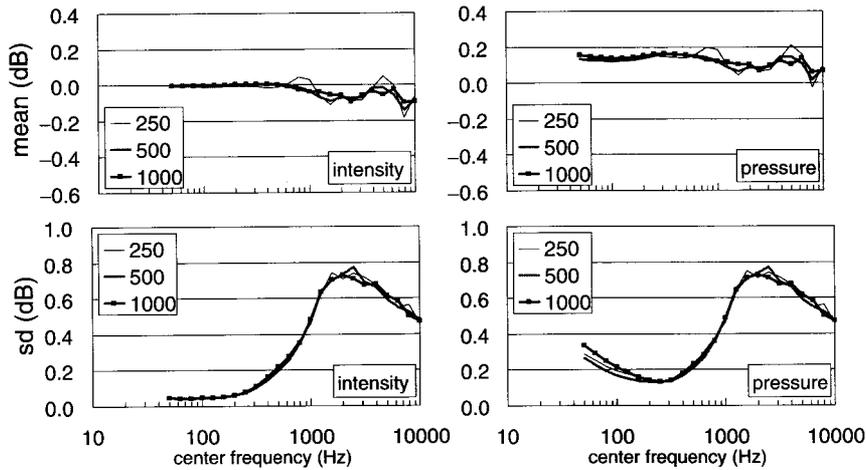


Fig. 5 The dependence of the means and standard deviations of errors on the number of calculation repetition (N).

statistical values are shown in Fig. 5. In the case of $N = 250$, a larger fluctuation of the mean value are observed compared to the cases with $N = 500$ and 1,000. The standard deviation is very insensitive to the number of repetitions.

5. THE COMPARISON OF MEASUREMENT POINT GEOMETRY DEFINED BY ISO 3745 AND ISO/DIS 3744

One example of the application of the proposed sound source model is the evaluation of the measurement point locations such as those recommended by ISO standards. Since the errors of a specified measurement method is calculated precisely, it is possible to compare the sets of measurement point locations in terms of the mean, standard deviation, maximum, and minimum values of errors. ISO 3745 (precision method by the sound pressure) defines a set of 20 measurement points in a hemi-anechoic room as shown in Fig. 2 (Table D.1 in ISO 3745). ISO/DIS 3744 (engineering method by the sound pressure) defines two sets of 20 measurement points in Table B.1 “for all noise sources” and in Table B.2 “for broadband noise sources.” Positions of the measurement points of the set B.1 and the set B.2 on the horizontal and vertical planes are shown in Fig. 6. A clear difference between these two measurement point types is that the points of the former set are randomly distributed, whereas, those of the latter set are very regularly distributed as those of the set D1.

Figure 7 shows the errors (mean, standard deviation, maxima, and minima) for the three sets of measurement positions. The parameters used for the calculations are shown in Table 1 (except for the measurement point geometry). The results of the mean errors show that the set B.1 has large negative errors in the range from 0.4 kHz to 2 kHz and the set B.2 has large errors from 1 kHz to

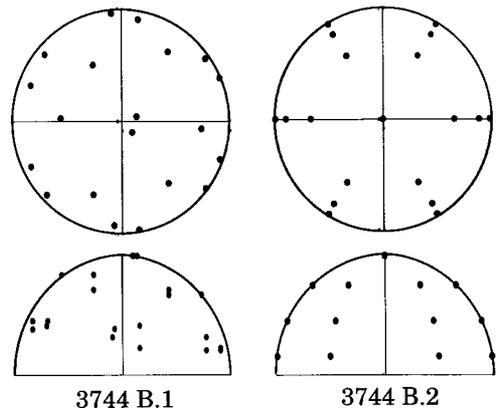


Fig. 6 The horizontal (top) and vertical (bottom) plane distributions of the twenty measurement points recommended by Table B.1 (left) and Table B.2 (right) both in ISO/DIS 3744 (2006).

2.5 kHz. On the other hand, the error of the set D.1 stays very low in the full range. The set B.2 has the smallest standard deviation of the errors especially below 1 kHz. The same trend is seen in the minimum errors. The set D.1 has a good overall performance.

The present source model combined with the statistical analysis method shown above provides us a convenient tool for the evaluation of measurement point distribution.

6. A PROPOSAL OF A NEW SET OF MEASUREMENT POINT DISTRIBUTION

Discussions in section 5 give us some hints for the improvement of the measurement point location. The present sound source model in 4π space is always

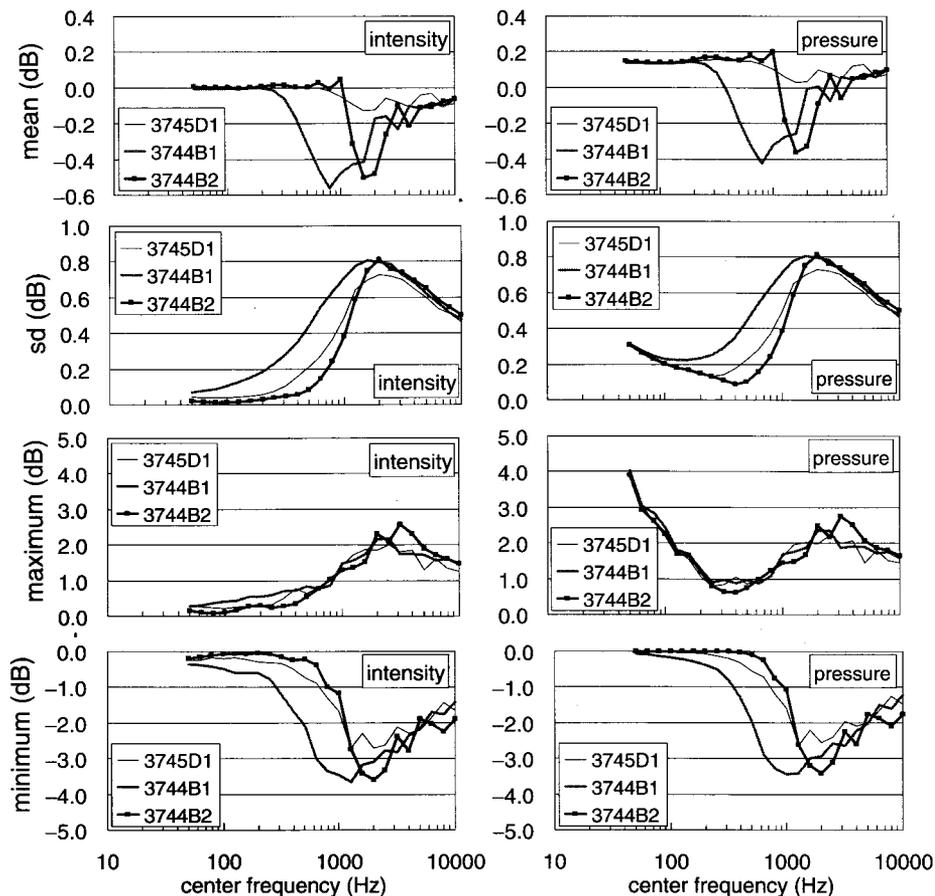


Fig. 7 The comparison of the errors (means, standard deviations, maxima, and minima) of the three sets of measurement point geometries D.1, B.1, and B.2.

symmetric with respect to the x - y plane because of the mirror image property (see Fig. 1). This contributes to the creation of large sound pressures along the x - y plane. The order of magnitudes of the mean errors of the sets B.1, B.2, and D.1 in Fig. 7 is the same as the order of the minimum heights of the three sets, which are $0.2R$ (2 points) for the set B.1, $0.15R$ (6 points) for the set B.2, and $0.025R$ (1 point) for the set D.1. The set D.1 is advantageous for the measurement methods on the infinitely large reflecting plane because it has a larger number of measurement points close to the floor. However, the set D.1 has one probable disadvantage that the distribution of the measurement points is very regular. This can be improved by rotating the points 7 to 12 by 30 degrees, points 13 to 18 by 60 degrees, and points 19 and 20 by 90 degrees (the numbering starts from the lowest point). The horizontal plane distribution of this proposed set is shown in the right-side chart of Fig. 2.

The present imaginary box has the largest dimension in

its height. The effect of the tallness of the sound source is enhanced by the mirror image. It seems important, therefore, to see the effects of the shape of the imaginary box. Figure 8 shows the comparison of errors between the set D.1 and the proposed measurement set. The sizes of the imaginary box were changed to $b_1 = 0.4$ m, $b_2 = 0.5$ m, and $b_3 = 0.3$ m. Other parameters are those shown in Table 1. We can see slight improvements in the standard deviation in the frequency range from 0.4 to 1.25 kHz.

Another possible shape of the imaginary sound source is a hemi-sphere instead of a box. In this case, the whole sound source including the image part is included within the radius of the "imaginary sphere," and the "tallness" of the image sound source is removed. The condition of ISO 3745 [2], "measurement radius must be greater than or equal to twice the maximum dimension of the box," is not satisfied. However, the condition of the hemi-spherical measurement surface of the ISO/DIS 3744 [1]; "the measurement radius shall be equal to or greater than twice

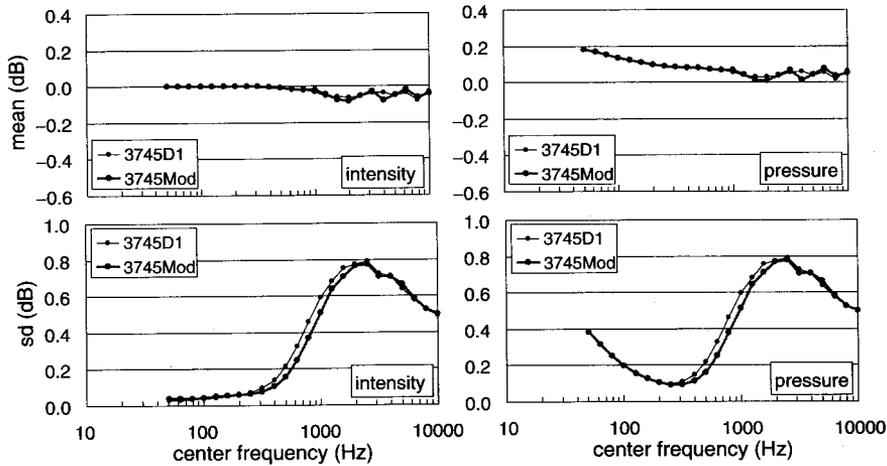


Fig. 8 The comparison of errors (means and standard deviations) obtained by using the two sets of measurement point geometries D.1 and the proposed set (Fig. 2, right). The sizes of the imaginary box are $b_1 = 0.4$ m, $b_2 = 0.5$ m, $b_3 = 0.3$ m.

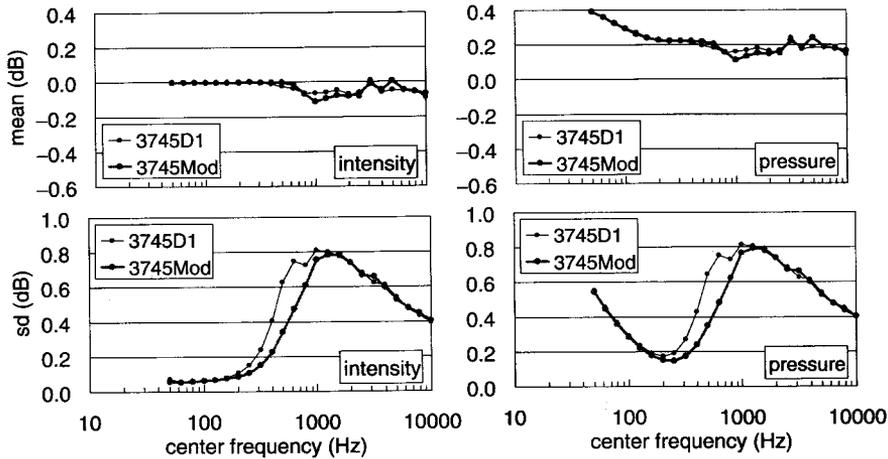


Fig. 9 The comparison of mean errors and standard deviations of errors obtained by using the two sets of measurement point geometries D.1 and the proposed set (Fig. 2, right). The point-like sources are randomly distributed inside an imaginary hemi-sphere with 0.5 m radius.

the characteristic source dimension, d_0 , and not less than 1 m and not greater than 16 m" is satisfied (in the present case, $d_0 = \sqrt{(b_1/2)^2 + (b_2/2)^2 + b_3^2} \leq 0.5$).

The results for the case of the hemi-sphere of radius 0.5 m are shown in Fig. 9 (other parameters are those shown in Table 1). The results clearly indicate that the proposed set is superior to the set D.1. The differences of the mean errors are very small but large improvements are seen in the standard deviations. The set proposed here is one distribution from many other possible distributions that have the potential to achieve better results than those existing in the present standards.

7. CONCLUSIONS

A multiple point-like source model was employed for the numerical evaluation of radiated power measurement errors using either the pressure or the sound intensity method. This approach is useful for this type of evaluation since a formula to determine the exact radiated power is available.

First, the effects of various parameters of the sound source model such as the number of point-like sources, the number of tonal components in 1/3 octave bands, and the number of calculation repetitions on the measurement

errors were evaluated, and some requirements on these measurement parameters were discussed. Second, the three sets of measurement point geometries recommended in ISO standards were compared. The results clearly showed quantitative differences of errors among the three sets, indicating that the sound source model proposed here is very useful for the evaluation of measurement point geometries. From the quantitative relations between the measurement point geometries and the measurement errors, a new set of measurement point geometry was proposed and an improvement of accuracy was confirmed.

There is a possibility that the proposed sound source model and statistical evaluation technique, if combined with methods such as Monte Carlo method, may result in an ideal or ultimate measurement point geometry.

REFERENCES

ISO/DIS 3744: 2006, Acoustics — Determination of sound power levels of noise sources using sound pressure — Engineering methods in an essentially free field over a reflecting

plane (2006).

- [2] ISO 3745: 2003, Acoustics — Determination of sound power levels of noise sources using sound pressure— Precision methods for anechoic and hemi-anechoic rooms (2003).
- [3] ISO 3746: 1995, Acoustics — Determination of sound power levels of noise sources — Survey methods using an enveloping measurement surface over a reflecting plane (1995).
- [4] ISO 9614-1: 1993, Acoustics — Determination of sound power levels of noise sources using sound intensity — Part 1: Measurement at discrete points (1993).
- [5] ISO 9614-2: 1996, Acoustics — Determination of sound power levels of noise sources using sound intensity — Part 2: Measurement by scanning (1996).
- [6] ISO 9614-3: 2002, Acoustics — Determination of sound power levels of noise sources using sound intensity — Part 3: Precision method for measurement by scanning (2002).
- [7] E. Skudrzyk, *The Foundations of Acoustics* (Springer-Verlag, New York, 1971).
- [8] M. Tohyama, H. Suzuki and Y. Ando, *The Nature and Technology of Acoustic Space* (Academic Press, London, 1995).
- [9] M. Nakamura, H. Suzuki and J. Tichy, "Accuracy evaluation of the sound power measurement by the sound pressure and the sound intensity methods," *Proc. VSTech 2005* (Hiroshima), pp. 123–126 (2005).